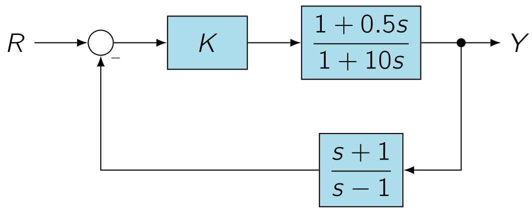
Control Systems : Set 6 : Loopshaping (2) - Solutions

Prob 1 | Use Matlab to draw the Nyquist plot for the system below. Determine the range of K for which the system is stable using the plot.



Note: In class we consider the loop-gain to be $K \cdot G$, but here we also have the dynamics of the sensor $\frac{s+1}{s-1}$ to consider. To decide what we should be plotting for our Nyquist diagram, we have to consider what the closed-loop system will be. Define $G = \frac{1+0.5s}{1+10s}$ and $H = \frac{s+1}{s-1}$, then transfer function from R to Y is

$$\begin{aligned} Y &= GK(R - HY) \\ Y(1 + KGH) &= GKR \\ \frac{Y}{R} &= \frac{KG}{1 + KGH} \end{aligned}$$

The Nyquist criterion is then asking the question: Does $1 + KGH$ have any zeros in the right-half plane? So to answer this question we plot the Nyquist plot of $L = KGH$ and test loop at how many encirlements we have of -1.

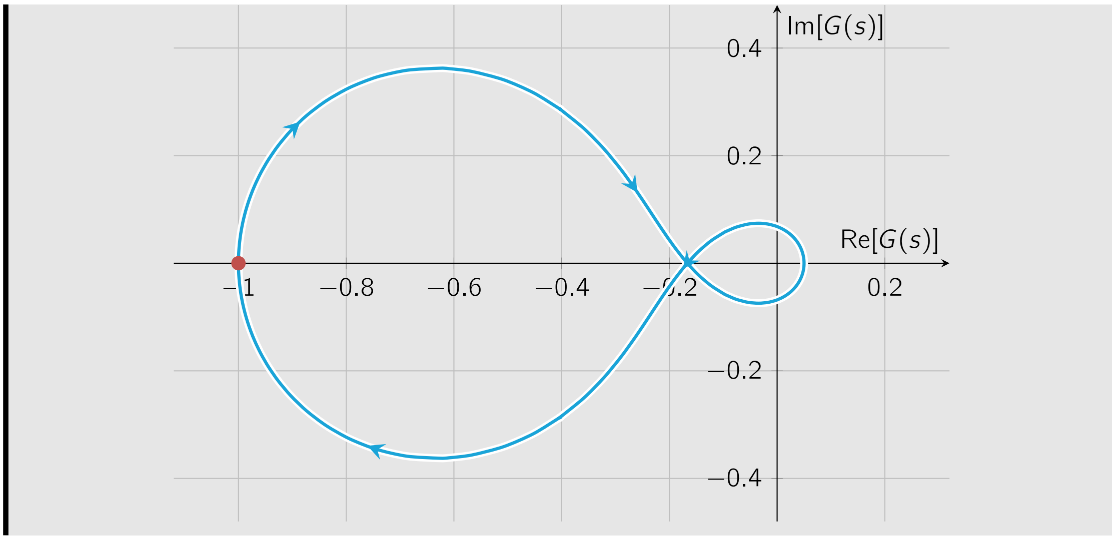
```
s = tf('s')
G = (1+0.5*s)/(1+10*s);
H = (s+1)/(s-1);

nyquist(G*H)
```

The system has one open-loop unstable pole, and so we need one right-to-left crossing for the closed-loop system to the stable.
From this we see that the closed-loop system will be stable if

$$K > 0 \text{ and } -1/K > -0.167 \quad \Rightarrow \quad K > 6$$

or

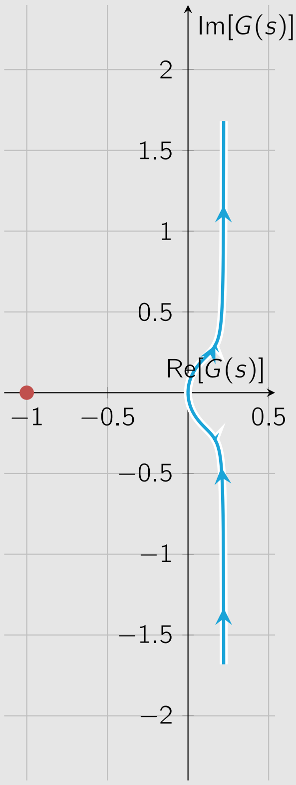
$$K < 0 \text{ and } -1/K < 0.05 \quad \Rightarrow \quad K < -20$$


Im[G(s)]
0.4
0.2
Re[G(s)]
−1
−0.8
−0.6
−0.4
−0.2
0.2
−0.2
−0.4

Prob 2 | Use Matlab to draw the Nyquist plot for

$$G(s) = \frac{K(s+1)}{s(s+3)}$$

and determine the range of K for which the system is stable using the Nyquist criterion.

> We see from the Nyquist plot below that the system will be stable for all positive values of K by the simplified Nyquist criterion.
>
> 
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